← não retirar o grampo RA:

PUC CAMPINAS
PONTIFÍCIA UNIVERSIDADE CATÓLICA

Sistemas de Controle e Servomecanismo.

Prof. Salles

Nome ______________________________ RA ______________ 13/04/2005

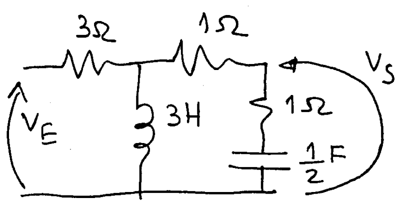
Leia a prova inteira. Sem consulta. Duração: 1h30min. Boa sorte !
Permitido o uso de calculadora. Dar as respostas com duas casas depois da vírgula.

<u>Escrever com caneta a resposta final</u>.

1. Para o sistema representado pelo circuito abaixo, onde a entrada é a tensão V_E e a saída é a tensão V_S , pede-se: 2,5

   a) determinar a função de transferência G(s)
   b) a resposta $V_S(t)$ para uma entrada V_E (t) = 3*u(t), onde u(t) é o degrau unitário — 2,5

obs.: condições iniciais nulas e t>0



$$V_S = \left(1+\frac{2}{s}\right)\cdot I_2$$

a

$$\begin{bmatrix} 3+3s & -3s \\ -3s & 2+3s+2/s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_E \\ 0 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 3+3s & V_E \\ -3s & 0 \end{vmatrix}}{\begin{vmatrix} 3+3s & -3s \\ -3s & 2+3s+\frac{2}{s} \end{vmatrix}} = \frac{3s V_E}{(3+3s)(2+3s+\frac{2}{s}) - (-3s)(-3s)}$$

$$\frac{V_S}{V_E} = \frac{(1+2/s)\cdot 3s}{(3+3s)(2+3s+\frac{2}{s}) - 9s^2} = \frac{3s+6}{15s+12+\frac{6}{s}} = \frac{s^2+2s}{5s^2+4s+2}$$

RASC

2	+3s	2/s
	3s	3
6	9s	6/s
$9s^2$ 6	6s	

b

$$\frac{s(s+2)}{s(s^2+\frac{4}{5}s+\frac{2}{5})}\cdot\frac{3}{s}$$

0,4899

1,9596

$$c(t) = \frac{3}{5}e^{-\frac{2}{5}t}\cos\left(\frac{\sqrt{6}}{5}t\right) + \frac{4\sqrt{6}}{5}e^{-\frac{2}{5}t}\operatorname{sen}\left(\frac{\sqrt{6}}{5}t\right)$$

← não retirar o grampo RA:

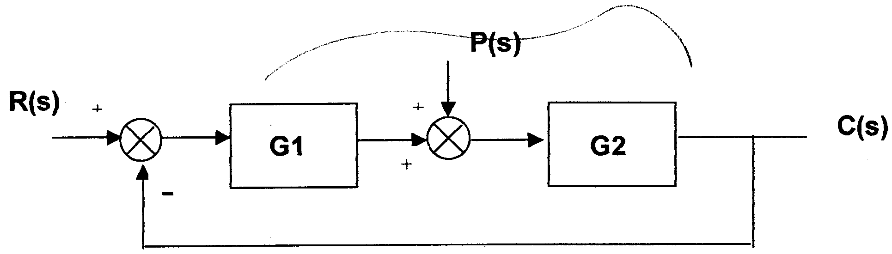
2. Para o sistema representado pelas funções de transferência abaixo, pede-se:
   a) a resposta c(t) do sistema sujeito a uma entrada r(t) = 3*u(t) onde u(t) é o "degrau unitário" e com uma perturbação nula (p(t) = 0). } 2,0
   b) a resposta c(t) do sistema sujeito a uma perturbação p(t) = $4*e^{-6*t}$ (exponencial decrescente, onde "e" é o número neperiano; e = 2,718...), sendo r(t) = 0 } 3,0

$$G1(s) = \frac{2s}{(s+3)}$$

$$G2(s) = \frac{1}{(s+2)}$$

realimentação unitária (H(s)=1)
obs.: condições iniciais nulas

$$G = \underbrace{\frac{2s}{s+3}}_{G1} \cdot \underbrace{\frac{1}{s+2}}_{G2}$$



$$\frac{G}{1+GH} = \frac{\frac{2s}{(s+2)(s+3)}}{1+\frac{2s}{(s+2)(s+3)}\cdot 1} = \frac{2s}{\underbrace{s^2+7s+6}_{(s+1)(s+6)}}$$

a)

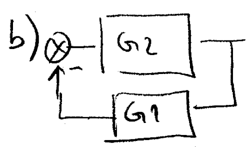
$$C(s) = \frac{2\cancel{s}}{(s+1)(s+6)} \cdot \underbrace{\frac{3}{\cancel{s}}}_{R(s)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+6}$$

$$c(t) = \frac{6}{5}e^{-t} - \frac{6}{5}e^{-6t}$$

cálculo de B

$A(s+6)^2 + B(s+1)(s+6)$
$+C(s+1)$
$= 12+3$

b)



$$\frac{\frac{1}{s+2}}{1+\frac{2s}{s+3}\cdot\frac{1}{s+2}} = \frac{(s+3)}{(s+1)(s+6)} \cdot \frac{4}{(s+6)} = \frac{A}{s+1} + \frac{B}{s+6} + \frac{C}{(s+6)^2}$$

$$c(t) = \underbrace{\frac{8}{25}}_{0,32}e^{-t} + \underbrace{\frac{12}{5}}_{2,4}te^{-6t} - \frac{8}{25}e^{-6t}$$

RAS	RASC	RASC	
$s+2$ $s+3$ $3s \quad 6$ $s^2 \; 2s$	$s = \frac{-7 \pm \sqrt{7^2-4\cdot1\cdot6}}{2\cdot1}$ $s = \frac{-7\pm5}{2}$ → -1, -6	$A = \frac{6s}{(s+1)(s+6)} = 0$ $B = \frac{6}{s+6}\Big\|_{s=-1} = \frac{6}{5}$ $C = \quad = \frac{-6}{5}$	$A = \frac{s+3}{(s+6)^2}\Big\|_{s=-1} = \frac{8}{25}$ $C = \frac{4(s+3)}{(s+1)}\Big\|_{s=-6} = \frac{12}{5}$

2ª Questão GABARITO

$$\frac{3(s+2)}{5s^2+4s+2} = \frac{3}{5}\left[\frac{s+2}{s^2+\frac{4}{5}s+\frac{2}{5}}\right]$$

$$= \frac{3}{5}\left[\frac{s+2}{\left(s+\frac{2}{5}\right)^2+\left(\frac{\sqrt{6}}{5}\right)^2}\right] = \frac{3}{5}\left[\frac{s+2/5 \quad + 8/5}{\left(s+\frac{2}{5}\right)^2+\left(\frac{\sqrt{6}}{5}\right)^2}\right]$$

$$c(t) = \frac{3}{5}\cdot e^{-\frac{2}{5}t}\cos\frac{\sqrt{6}}{5}t \quad + \frac{8}{\sqrt{6}}\cdot\frac{3}{5}\cdot\frac{\sqrt{6}}{5}\, e^{-\frac{2}{5}}\operatorname{sen}\frac{\sqrt{6}}{5}t$$

$$\frac{\overset{4}{\cancel{24}}\sqrt{6}}{5\cdot\cancel{6}_1} = \frac{4\sqrt{6}}{5}$$

$$s^2+\frac{4}{5}s+\frac{2}{5}$$

$$s = \frac{-\frac{4}{5}\pm\sqrt{\left(\frac{4}{5}\right)^2-4\cdot1\cdot\frac{2}{5}}}{2\cdot1} = \frac{-\frac{4}{5}\pm\sqrt{\frac{16}{25}-\frac{8\cdot5}{5\cdot5}}}{2}$$

$$s = \frac{-\frac{4}{5}\pm\frac{1}{5}\sqrt{16-40}^{\,24}}{2} = \frac{-\frac{4}{5}\pm\frac{2}{5}\sqrt{6}}{2} = -\frac{2}{5}\pm\frac{\sqrt{6}}{5}$$

24	2
12	2
6	2
3	3
1	

$2\sqrt{6}$

Pares de transformada de Laplace

	$f(t)$	$F(s)$
1	Impulso unitário $\delta(t)$	1
2	Degrau unitário $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{s^n}$
5	e^{-at}	$\frac{1}{s+a}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$\frac{1}{(n-1)!}t^{n-1}e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
8	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
9	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$
10	$\text{sen}\,\omega t$	$\frac{\omega}{s^2+\omega^2}$
11	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
12	$e^{-at}\,\text{sen}\,\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
13	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
14	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\,\text{sen}\,\omega_n\sqrt{1-\zeta^2}\,t$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
15	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\,\text{sen}\,(\omega_n\sqrt{1-\zeta^2}\,t - \phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
16	$1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\,\text{sen}\,(\omega_n\sqrt{1-\zeta^2}\,t + \phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$

Propriedades das transformadas de Laplace

1	$\mathcal{L}_\pm\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
2	$\mathcal{L}_\pm\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$
3	$\mathcal{L}_\pm\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}\overset{(k-1)}{f}(0\pm)$ onde $\overset{(k-1)}{f}(t) = \frac{d^{k-1}}{dt^{k-1}}f(t)$
4	$\mathcal{L}_\pm\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)\,dt\right]_{t=0\pm}$
5	$\mathcal{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$
6	$\int_0^\infty f(t)\,dt = \lim_{s\to 0} F(s) \quad$ se $\int_0^\infty f(t)\,dt$ existir
7	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$
8	$\mathcal{L}[f(t-a)1(t-a)] = e^{-as}F(s) \qquad a \geq 0$
9	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$
10	$\mathcal{L}[t^2f(t)] = \frac{d^2}{ds^2}F(s)$
11	$\mathcal{L}[t^nf(t)] = (-1)^n\frac{d^n}{ds^n}F(s) \qquad n = 1, 2, 3, \ldots$
12	$\mathcal{L}\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s)\,ds \quad$ se $\lim_{t\to 0}\frac{1}{t}f(t)$ existir
13	$\mathcal{L}\left[f\left(\frac{t}{a}\right)\right] = aF(as)$
14	$\mathcal{L}\left[\int_0^t f_1(t-\tau)f_2(\tau)\,d\tau\right] = F_1(s)F_2(s)$
15	$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(p)G(s-p)\,dp$